national accelerator laboratory

NAL-Conf-73/56-THY

August 1973

A Survey of Developments in Strong Interaction Theories*

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*Invited paper presented at the 1973 meeting of the Division of Particles and Fields of the APS, Berkeley, California, August 1973.

I APOLOGIES AND INTRODUCTION

Strong interaction theories cover a broad and interesting range of ideas and phenomena. Although we all in our logical hearts expect that we will someday know from the same physical theory the explanation of why, say, the observed baryons come in octets and decimets <u>and</u> why the intercept of secondary Regge trajectories have $\alpha(0) \approx 1/2$, at this point in time the views on these and other topics are at best fragmented. In order to present today a finite discussion of some ideas in hadronic physics, I have been forced to choose among a large selection of fascinating subjects. I propose, then, to address three questions during this talk:

1. Rising total cross sections and theories which attempt to understand this.

2. Gauge theories of strong interactions and the possible relevance of such field theories to scaling.

3. Dual String Models

Obviously I have left out vast numbers of worthwhile topics; I trust that my distinguished colleagues speaking in other sessions will cover them in depth.

II. RISING TOTAL CROSS SECTIONS

There can be no doubt that the most interesting new phenomenon to have been observed in hadronic physics during the last year is the $\sim$ 10% rise in the proton proton total cross section over the range of CERN-IRS energies,[1] $400\ (\mathrm{GeV})^2 \leq s \leq 3000\ (\mathrm{GeV})^2$. I show a casual picture of this in Fig. 1 which I borrowed from the nice review talk of M. Jacob.[2]

Two basic views of this rise in σ_T^{pp} can be taken:

(a) It is an indication of a real trend for σ_T to continue to rise probably saturating the Froissart bound. This is supported primarily by the "eikonal school" although occasionally a bootstrap theorist is found in this corner.

(b) The rise in σ_T is a transient phenomenon. At larger energies (alas, not yet available) σ_T will settle into its real asymptotic behavior which is

(i) σ_T decreases as a very small power of s or

(ii) σ_T eventually goes to a constant.

Let's discuss these ideas in order. The theories which yield σ_T saturating the Froissart bound usually begin with the eikonal form of the elastic scattering amplitude[3]

$$T_{el}(s,t) = is\int d^2b\ e^{i\underset{\sim}{\Delta}\cdot\underset{\sim}{b}}\left[e^{i\chi(s,\underset{\sim}{b})}-1\right], \tag{1}$$

where $\underset{\sim}{\Delta}$ is a two vector momentum transfer in the x, y plane when the beam comes in along the z-axis; $t = -|\underset{\sim}{\Delta}|^2$. $X(s, \underset{\sim}{b})$ is the eikonal phase as a function of the energy, s, and the impact parameter $\underset{\sim}{b}$. X, which has the dynamics in it, is calculated from some "born graphs" or some infinite set of Feynman graphs.[4] In quantum electrodynamics if one takes certain sets of tower graphs (see Fig. 2) then for fixed $\underset{\sim}{b}$, $X(s, \underset{\sim}{b})$ behaves as

$$X(s, \underset{\sim}{b}) \sim s^{1+\epsilon}, \quad \epsilon > 0 \tag{2}$$

and the b dependence is rather like

$$X(s, \underset{\sim}{b}) \sim e^{-\mu b} \tag{3}$$

where μ is some characteristic mass. The T-matrix as a function of s and b then is very close to a Θ function

$$e^{iX(s,\underset{\sim}{b})} - 1 = T_{el}(s, \underset{\sim}{b}) \approx \Theta\left(r_o \log s - |\underset{\sim}{b}|\right), \tag{4}$$

and this gives

$$T_{el}(s,t) = 2\pi i\, s\, (r_o \log s)^2 \left[\frac{J_1(\sqrt{-t}\, r_o \log s)}{\sqrt{-t}\, r_o \log s}\right], \tag{5}$$

so

$$\sigma_T(s) = \pi r_o^2 (\log s)^2 . \tag{6}$$

This exactly saturates the Froissart bound

$$\sigma_T(s) \le (\log s)^2 , \tag{7}$$

and is to be thought of as the leading approximation to an expansion of $\sigma_T(s)$ in decreasing powers of log s. Indeed according to Jacob[2] a fit to the pp data can be achieved by

$$\sigma_T(s) = 38\text{mb} + 0.68\text{mb}\,\log^2\left(\frac{p_{\text{lab in GeV/c}}}{100}\right). \qquad (8)$$

Some comments are in order. First, it must not be thought that this $(\log s)^2$ is a firm prediction of quantum field theory. The specific form for the eikonal phase is a choice abstracted from a summation of leading behaviors of selected Feynman graphs - if σ_T does not behave as $(\log s)^2$, one does not throw away field theory - only perhaps some field theorists. Second, in these same models the average multiplicity $\bar{n}(s)$ of produced particles grows as a small power times log s. Third, because the leading singularity in the J-plane is more complicated than a pole, one expects long range correlations yielding integrated correlation functions $f\,k\,(s)$ which behave as

$$fk(s) \sim (\log s)^k \qquad (9)$$

and finally, in these models the diffraction peak in t of $\frac{d\sigma_{\text{elastic}}}{dt}$ shrinks as $(\log s)^2$ as one sees directly from Eq. (5). This last point is not a feature of present pp elastic data.[2]

The same structure for $T_{el}(s,t)$ comes from a marriage of the absorption model with the multiperipheral model.[5] Of course, the same critical comments apply if one substitutes for "abstractions from field theory" the phrase "conjectures from a presumed self consistent theory of diffraction scattering."

To saturate the Froissart bound and have total cross sections rising slowly forever has a definite aesthetic appeal. The details of the theories which yield $\sigma_T \sim (\log s)^2$ are less appealing. One must deal with the observations made above, especially the shrinkage of the elastic diffraction peak, and one must provide some understanding for the unusually large energies at which the $(\log s)^2$ growth sets in; note the scale setting factor of 100 GeV/c in Eq. (8).

Now to the view that the rise in σ_T is a transient phenomenon. Basically there are two schools of thought. The first says that what we are seeing is just the effect of two or more Pomeron cut exchange (Fig. 3) which is emerging as the contributions of secondary Regge trajectories with $\alpha(0) = 1/2$ disappears into the noise. One would expect $\sigma_T(s)$ to take the form

$$\sigma_T(s) = \sigma_0\left(1 - \frac{a}{b+\log p_{lab}}\right) + 0\left(\frac{1}{\sqrt{p_{lab}}}\right) \quad . \tag{10}$$

Indeed, a fit to the ISR data of the form[2]

$$\sigma_T = 60\text{mb}\left(1 - \frac{3\text{mb}}{3+\log(p_{lab}\text{ in GeV/c})}\right) \quad , \tag{11}$$

seems acceptable, if not compelling. From either a theoretical or a phenomenological point of view there is little one can say to fault this attitude. It is true that no one has yet produced a consistent, well formulated theory of diffraction scattering which yields (10) without having other ghastly features.[6]

The second school of thought attributes the rise in $\sigma_T(s)$ to the contribution of the triple Pomeron coupling in T_{el}.[7] The triple Pomeron coupling is thought to give a contribution to $\sigma_T(s)$ of the form[8]

$$\sigma_{Tp}(s) = \frac{g_p(0)}{16\pi\alpha'} \log\left(1 + \frac{2\alpha'}{b}\log s\right) \tag{12}$$

where α' is the slope of the Pomeron trajectory $\alpha p(t) = 1+\alpha' t$, $g_p(0)$ is the triple Pomeron coupling appropriately normalized, and b is a slope parameter associated with $g_p(t)$

$$g_p(t) = g_p(0)e^{bt}. \tag{13}$$

It is argued that when $2\alpha'/b \log s$ is small, then the triple Pomeron formula gives an effective log s rise to σ_T and later flattens off to the essentially irrelevant log (log s).

The major question associated with this explanation of the rise in σ_T would appear to be numerical. In order for this argument to be viable

$$\frac{2\alpha'}{b}\log s \ll 1 \tag{14}$$

If $\alpha' \approx 1/2$ and b is a fairly normal slope of 10 in $(\mathrm{GeV})^{-2}$, then we may use this idea for

$$\log s \leq 10, \tag{15}$$

which does cover the ISR range. However, to match the 4mb rise in σ_T as log s varies from about 4 to 8, one needs quite a sizeable $g_p(0)$. So large, in fact, that it is likely to be inconsistent with the values of $g_p(0)$ one infers from the peaking of the X distribution in $pp \rightarrow p + \text{anything}$ near $X = 1$ at ISR energies[2] - a phenomenon which is generally considered to be the only convincing evidence for the existence of a triple Pomeron coupling.

Numerical questions aside one may raise theoretical eyebrows at the use of the triple Pomeron. The original reaction to the log (log s) growth in σ_T arising from the triple Pomeron coupling was that it was not a virtue, but required $g_p(0)$ to vanish.[8] This led to a variety of disasters about vanishing Pomeron couplings, the most striking of which was the decoupling of the Pomeron from total cross sections themselves,[9] thus removing the rationale for a Pomeron with $\alpha(0) = 1$ in the first place. By taking an attitude that makes a virtue out of disaster, it seems to me that one has really postponed the difficult and interesting issue of how $\sigma_T(s)$ really behaves. Only the Berkeley school which advocates $\alpha(0) < 1$ has a logical answer to the hard question. Eventually σ_T must go to zero as a small power after the amusing, but transient, effect of the triple Pomeron rise wears itself out.

Another objection altogether is raised by Blankenbecler and his collaborators[3] who argue that the triple Pomeron contribution to the Feynman graph like Fig. 4, say Fig. 5, contributes not + g_p log (log s) to σ_τ but -g_p log (log s) and, never mind orders of magnitude or detailed fits, this has the wrong sign to explain a rise in σ_τ. This objection harks back to the ancient controversy over the sign of the two Pomeron cut to which the triple Pomeron piece of T_{el} is one contribution. The Feynman graph view says that when one takes Im T_{el} from the Y-graph of Fig. 5 to find its contributions to σ_τ, only one of these, Fig. 6a, is connected with the triple Pomeron coupling usually measured in inclusive processes. There are two other cuts of Fig. 5 which contribute to Im T_{el} looking Fig. 6b and they yield an amount -2g_p log (log s) to σ_τ.

Let's look into this argument a bit more. It is clear that we cannot stop with the Y-graph of Fig. 5 since we will eventually have a negative σ_τ. So we must sum up whole sets of contributions to Im T_{el} to find a positive σ_τ. Call $\tilde{g}_p$ the triple Pomeron coupling defined by Fig. 5; it is a coupling in a model only and isn't the physical g_p measured in inclusive reactions. Call β the elastic coupling coming, say, from the ordinary ladder, as in Fig. 7. Now representing the ladders by a wiggly line (bare Pomeron) consider the sum of sets of graphs as in Fig. 8. The graphs of Fig. 8a give with the "minus sign rule" a contribution to σ_T of

$$\sigma_T(s) \sim \frac{-\beta^3 g_p \log(\log s)}{1+g^2 \log(\log s)} . \tag{16}$$

Those of Fig. 8b give

$$\sigma_T(s) \sim \beta^2/1+g_p^2 \log(\log s) , \tag{17}$$

and from Fig. 8c we have

$$\sigma_T(s) \sim \frac{\beta^4 g_p^2 [\log(\log s)]^2}{1+g_p^2 \log(\log s)} . \tag{18}$$

Adding these contributions we find a net σ_T from triple Pomeron diagrams to be positive for large log (log s) and to behave as

$$\sigma_T \sim \beta^4 \log(\log s). \tag{19}$$

Does one infer then that $\beta=0$, since to produce log (log s) even in the simplest diagram of Fig. 8 we had to require $\alpha(0) = 1$ for the input ladder? I think not. Probably one is forced to the conclusion that without some rather more consistent theory of diffraction, the Feynman graph argument only dents but does not yet destroy the triple Pomeron couplers. That they have enough trouble on their own, we have already pointed out.

I think that a fair conclusion from the foregoing discussion is that the origin of the rise in σ_T and its non-transient implications for hadronic theories remains an open and fascinating question.

III. GAUGE THEORIES FOR STRONG INTERACTIONS

What I wish to discuss in this section are ideas which involve a certain level of speculation but are so exciting that no one ought to ignore them. As a fallout of the intense interest focused on renoralizable gauge theories of the weak and electromagnetic interactions it has been suggested that non-Abelian gauge field theories, of which the ancient Yang-Mills theory is a classical example, may also be relevant to hadronic physics.[10] In particular a rather recent development involving detailed properties of the renormalization group for these non-Abelian gauge theories has raised the amusing prospect[11] that one may be able to understand the scaling properties observed in deep inelastic electron scattering at SLAC.

It is not my purpose to turn us all into giant experts on the renormalization group in twenty minutes, but to briefly draw attention to the new ideas which are around and some problems associated with them.

What the renormalization group enables one to do is give a compact discussion of the behavior of various Green functions as the momenta involved approach large spacelike values. Then there are constraint equations on the Green functions which were derived by Callan and Symanzik[12] several years ago. These express how the masses and coupling constants of the field theory presumed to govern the interactions underlying the Green function may be varied.

A typical equation of this form reads

$$\left[m \frac{\partial}{\partial m} + \beta(g)\frac{\partial}{\partial g} - n\gamma(g)\right] G^{(n)}_{Asy}(p_i) = 0, \tag{20}$$

where m and g are the renormalized mass and coupling constants of the field theory, $\beta(g)$ and $\gamma(g)$ are finite functions of g alone and $G^{(n)}_{Asy}$ is the asymptotic form of an n-point Green function. The functions β and γ are known only in perturbation theory for all but the most special models.

The key observation is that zeroes of $\beta(g)$ where the slope of β is <u>negative</u> govern the asymptotic behavior of $G^{(n)}$. (See Fig. 9). For example, if $\beta(g)$ has a simple zero at g_o and $d\beta/dg|_{go} < 0$, then for all momenta in $G^{(n)}$ going simultaneously to space-like infinity $p_i = \xi k_i$, $\xi \to \infty$, $G^{(n)}$ behaves as

$$G^{(n)}_{Asy} \sim \xi^{4-n} \times \xi^{-n\gamma(g_o)} \times f^{(n)}(g_o), \tag{21}$$

where ξ^{4-n} is just reminding us of the normal dimensions of $G^{(n)}$, $\xi^{-n\gamma(g_o)}$ is a measure of the "anomalous dimensions" due to the renormalization procedure, and $f^{(n)}(g_o)$ is some finite function of g_o. If $\gamma(g_o)$ were zero by some miracle, then there would be no anomalous dimensions and the theory would be scale invariant: that is, all quantities have only naïve dimensions.

Very little is known in general about the zeroes of $\beta(g)$. By its definition it turns out that $\beta(0) = 0$, but for almost all renormalizable field theories $d\beta/dg\big|_{g=0} > 0$. So other zeros, wherever they may be govern the asymptotic behavior of the Green functions. Furthermore, the functions $\gamma(g)$ will, most likely, not also vanish at these zeroes of $\beta(g)$ and such theories will not reproduce the scaling behavior seen at SLAC; indeed, they will deviate from scaling by powers of q^2.

Now for the good news. In a large class of non-Abelian gauge theories it has been found that[11]

$$\beta(g) = -\beta_0 g^3 + 0(g^5), \ \beta_0 > 0. \tag{22}$$

This means that the value of the couplings of the underlying field theories which govern the asymptotic behavior of Green functions is zero coupling. That is, free field theories give the asymptotic behavior and corrections to those asymptotic values are calculable by perturbation theory. Furthermore, for free field theories $\gamma(g)$ vanishes since there are no anomalous dimensions for a free field.

The detailed results of these renormalization group studies are that there are still logarithmic deviations from exact scaling in most instances and in general the approach to scaling is only logarithmic. For example, it is still true that for an underlying field theory of vector bosons (the gauge fields) and fermions the ratio $R = \sigma_L/\sigma_T$ of longitudinal over transverse photon cross sections for deep inelastic electron scattering goes to zero in the Bjorken limit but

$$\lim_{\substack{q^2 \to \infty \\ X \text{ fixed}}} R(X = \frac{-q^2}{2m\nu},\ q^2) \sim \frac{1}{\log q^2}. \tag{23}$$

And in electron-positron annihilation one finds

$$\frac{\sigma_{e^+e^- \to \text{hadrons}}}{\sigma_{e^+e^- \to \mu^+\mu^-}} \underset{q^2 \to \infty}{\sim} \sum_i Q_i^2 \left(1 + 0\left(\frac{1}{\log q^2}\right)\right), \tag{24}$$

where q^2 is now the energy of the e^+e^- system and Q_i are the charges of the constituents of the hadrons.

Of course, these theories are not without their problems. First of all, they are stuck, so far, on logarithmic approaches to scaling. If this is verified by the muon scattering experiments at NAL or by e^+e^- annihilation experiments at SPEAR, one will, of course, not regard this as a problem. Second, in order to give the vector bosons entering these theories some non-zero mass, one conventionally introduces some scalar (Higgs) bosons which develop non-zero vacuum expectation values, break the underlying gauge symmetry, and yield up masses for the gauge bosons. So far it has not been possible to carry this out completely without destroying the key result that $\beta'(g=0) < 0$. So activists in gauge theories have pinned their hopes on another mechanism, invented by Coleman and E. Weinberg,[13] which takes advantage of the infrared singular behavior of theories involving massless bosons and fermions to dynamically break the gauge symmetry and produce masses for the particles in the physical spectrum of the initial lagrangian. There is also the hope that the infrared behavior is so singular that the physical spectrum resulting from solving the bound state problem will not resemble the underlying fields ("quarks" and bosons)and one will not be faced with the usual embarrassment of "keeping the quarks in." Because of our less than dramatic success in solving the bound state problem in less singular field theories, one must view this as a strong hope indeed. Finally, there is the problem of which gauge group to use for the basic field theory. Hints from experiment will only come as the detailed nature of the predicted logarithmic approach to scaling is seen; that is, the powers of $\log q^2$ can yield up significant information. Clearly, however, the practical problems are quite non-trivial. Nevertheless, I regard this whole scheme as an attractive prospect for really learning something concerte about hadronic interactions; whether we will at the same time learn all we want to know, I hesitate to conjecture.

IV. DUAL STRING MODELS

The third and final subject I propose to consider today is that of the string picture in dual theories. You will recall that the spectrum of the dual resonance model was noted some years ago by Nambu and Susskind[14] to be identical to the excitation spectrum of a string in one dimension at every point of which there is attached a four vector $X_\mu(\sigma, \tau)$ as the actual space-time displacement vector of the constituents of the hadron-infinite in number and thus continuously distributed in σ. To reproduce the dual spectrum is then the requirement that $X_\mu(\sigma,\tau)$ obey the wave equation

$$\left(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2}\right) X_\mu(\sigma, \tau) = 0. \qquad (25)$$

To achieve this one builds an action out of X_μ and its derivatives and imposes constraint conditions reflecting the invariances of the action, if any, so that the classical Euler-Lagrange equations of motion for the independent components of X_μ are just (25).

The action which has been most thoroughly studied is that proposed by Nambu.[14] in which it is taken to be proportional to the invariant two dimensional area swept out by the string as it extends in σ and moves in time τ. This action is

$$A = -\frac{1}{2\pi\alpha'} \int_0^1 d\sigma \int_{\tau_i}^{\tau_f} dt \left\{\left(\frac{\partial X_\mu}{\partial\tau}\frac{\partial X^\mu}{\partial\sigma}\right)^2 - \left(\frac{\partial X_\mu}{\partial\sigma}\right)^2\left(\frac{\partial X_\mu}{\partial\tau}\right)^2\right\}^{\frac{1}{2}}, \qquad (26)$$

where the constant is conventional and the "length" of the string in parameter space has been chosen to be one. (See Fig. 11). The most careful study of this action has been by P. Goddard, et al.[15] Noting the invariance of the action under the general coordinate transformations

$$\sigma \rightarrow f(\sigma, \tau), \tag{27}$$

and

$$\tau \rightarrow g(\sigma, \tau), \tag{28}$$

they choose as auxiliary conditions on X_μ

$$\frac{\partial X_\mu}{\partial \sigma} \cdot \frac{\partial X^\mu}{\partial \tau} = 0, \tag{29}$$

and

$$\left(\frac{\partial X_\mu}{\partial \sigma}\right)^2 + \left(\frac{\partial X_\mu}{\partial \tau}\right)^2 = 0. \tag{30}$$

These guarantee that the independent components of X_μ satisfy (25).

When one is willing to work in d space-time dimensions, then the two fold invariance of the action means that precisely d-2 components of the space-time displacement vector $X_\mu(\sigma, \tau)$ are independent. This has the consequence that the first excited state created from the vacuum by the normal mode operators of X_μ has only d-2 degrees of freedom, and since this is one less than the normal number (one degree of freedom always being lost essentially through a mass-shell constraint), the first excited state must be massless. In terms of the Regge intercept of the leading trajectory of the dual spectrum, this implies $\alpha(0) = 1$, and further that the ground state (vacuum) is a tachyon with $\alpha' m^2 = -1$; α' is the universal slope of the dual model. All theories based on an action like (26) with "too much" symmetry suffer from this disease.

More detailed analysis shows that when one takes this action very seriously and requires that the geometrical generators of the Lorentz group which seems manifest in Eq. 26 have the commutation relations required by Lorentz symmetry, then there is a restriction on the number of space-time dimensions of the theory to d = 26. Physics, one recalls, takes place in d = 24.

There are two attitudes to take at this point: (1) the model action is a disaster and must be replaced by one with less symmetry or (2) the model action with all its faults is an interesting prototype theory for what a "real" theory in four dimensions must look like and deserves further intensive study. I think both points of view are correct in the usual spirit of inquiry. Clearly one needs a better action; nevertheless one may use (26) to learn a great deal about any future, more realistic string models.

The most significant contribution to this latter point of view has been by Mandelstam.[16] By studying the functional integral formulation of the quantum theory of the string with (26) as a classical action, he is able to reproduce the n-point amplitudes of the dual resonance model and to justify the attractive, if picturesque language, of imagining strings in space-time coming together and splitting. The basic dual vertex consists of one string splitting into two (Fig. 12) and higher point functions are similarly described. Mandelstam is also able to explicitly demonstrate the role that space-time dimension 26 plays in guaranteeing the Lorentz covariance of the theory.

Although the string model is as yet rather an unphysical entity (tachyons and d=26 just smell peculiar), it still holds an attractive allure and both ought and will be the subject of study for its improvement, perhaps to a physically realistic model.

ACKNOWLEDGMENTS

I am very grateful to R. J. Crewther, M. B. Einhorn, S. D. Ellis and S. Nussinov for patiently helping me to learn about the subjects covered in this brief survey.

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FIGURE CAPTIONS

Fig. 1 A representation of the Serpukhov, NAL, and CERN-ISR data on the proton-proton total cross section.

Fig. 2 Tower graphs summed in quantum electrodynamics to give an eikonal phase $X(s,\underset{\sim}{b})$ which grows as $s^{1+\epsilon}$, $\epsilon>0$.

Fig. 3 The Pomeron pole and absorptive Pomeron cuts which lead to total cross sections which rise logarithmically to a constant.

Fig. 4 The triple Pomeron contribution to the elastic scattering amplitude. The contribution of this to the total cross section is proportional to log (log s).

Fig. 5 A Feynman graph representation of the triple Pomeron amplitude.

Fig. 6 Two cuts of Fig. 5 which contribute to Im T_{el} and, therefore, σ_{τ}.

Fig. 6a Yields the inclusive cross section near the edge of phase space and thus the usual triple Pomeron result.

Fig. 6b Another cut which in the eikonal model serves to change the sign of Fig. 6a.

Fig. 7 A ladder graph representation of the $t = 0$ Pomeron exchange with factorizable residue β^2.

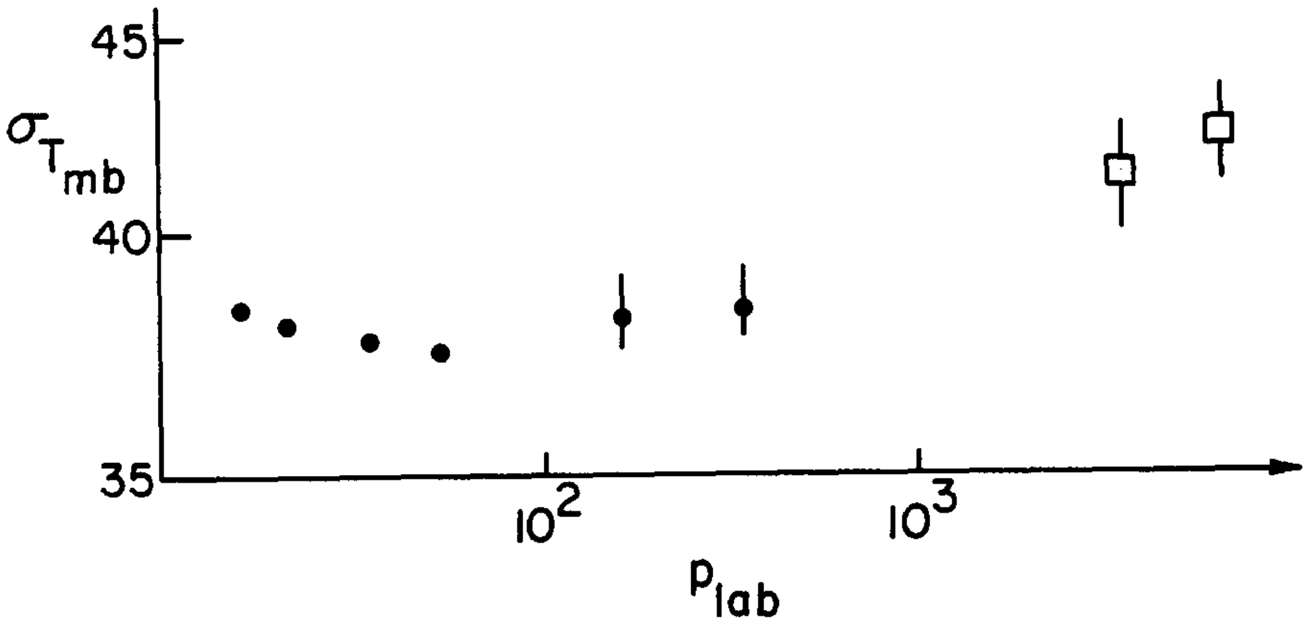
Fig. 8 Various sets of Reggeon graphs which are summed up in the text with the eikonal sign for Pomeron cuts.

Fig. 9 The behavior of the coupling constant function $\beta(g)$ in most renormalizable field theories and in many non-Abelian gauge theories. The zeroes of $\beta(g)$ where $\beta' < 0$ govern the asymptotic behavior of Green functions in field theory.

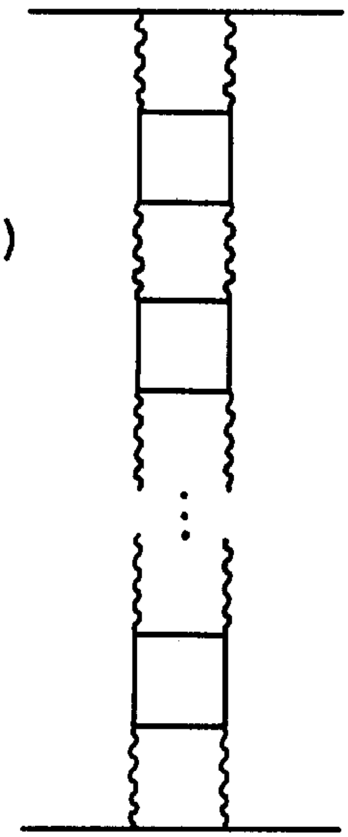
Fig. 10 A picture of the one dimensional string stretching from 0 to 1 in the σ parameter space. A vector $X_\mu(\sigma, \tau)$ is attached to each point σ at each time τ. It is supposed to represent the displacement in real space-time of the string taken to be a hadron.

Fig. 11 The area between $0 \le \sigma \le 1$, $\tau_i < \tau_f$ swept out by the hadronic string. The classical action, Eq. 26 is chosen to be proportional to this area.

Fig. 12 One string (hadron) splits into two strings (hadrons) to form the basic dual vertex.

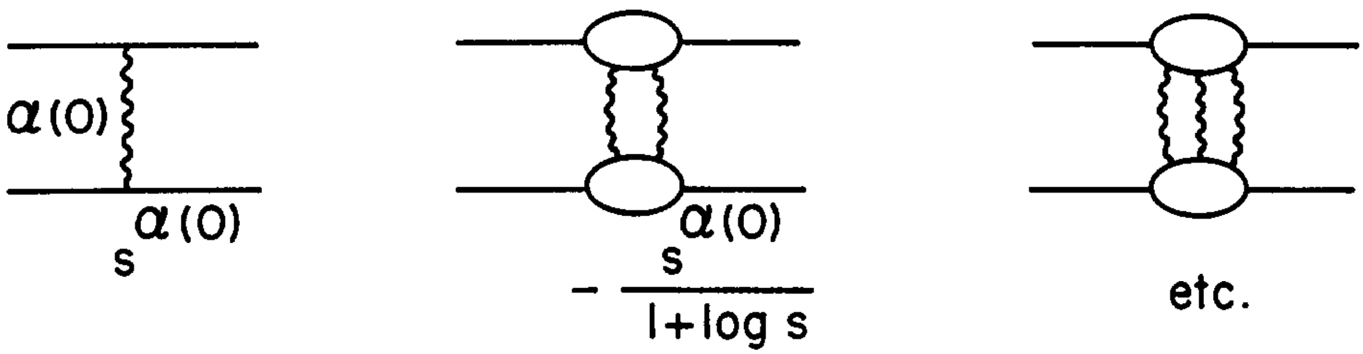

45
$\sigma_{T\,mb}$
40
35
10^2
10^3
p_{lab}

$X(s, \underset{\sim}{b})$



$\sim s^{1+\epsilon}$

$\epsilon > 0$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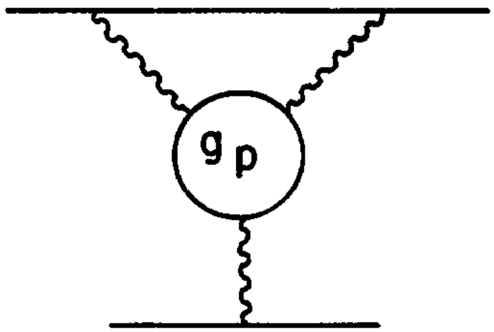
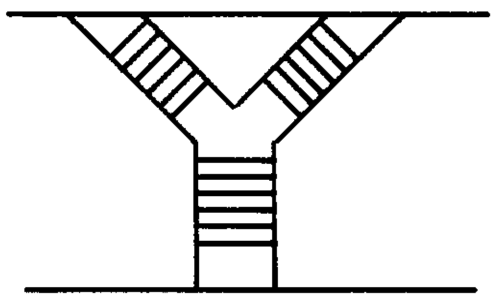
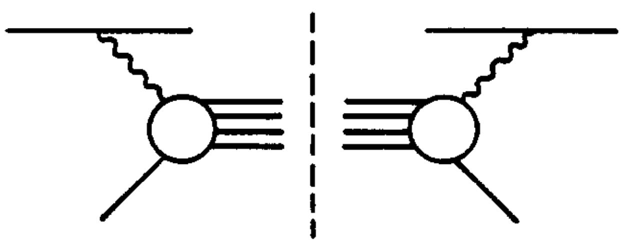
s α(0)
s α(0)
1+log s
etc.

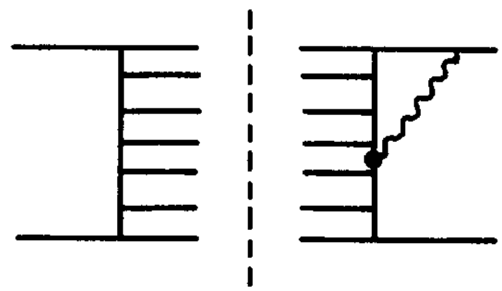


$$\sigma_{Tp} \sim \log\left(1 + \frac{2\alpha'}{b} \log s\right)$$

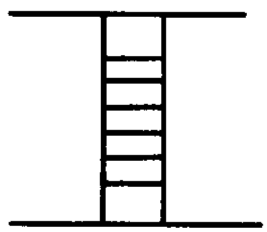




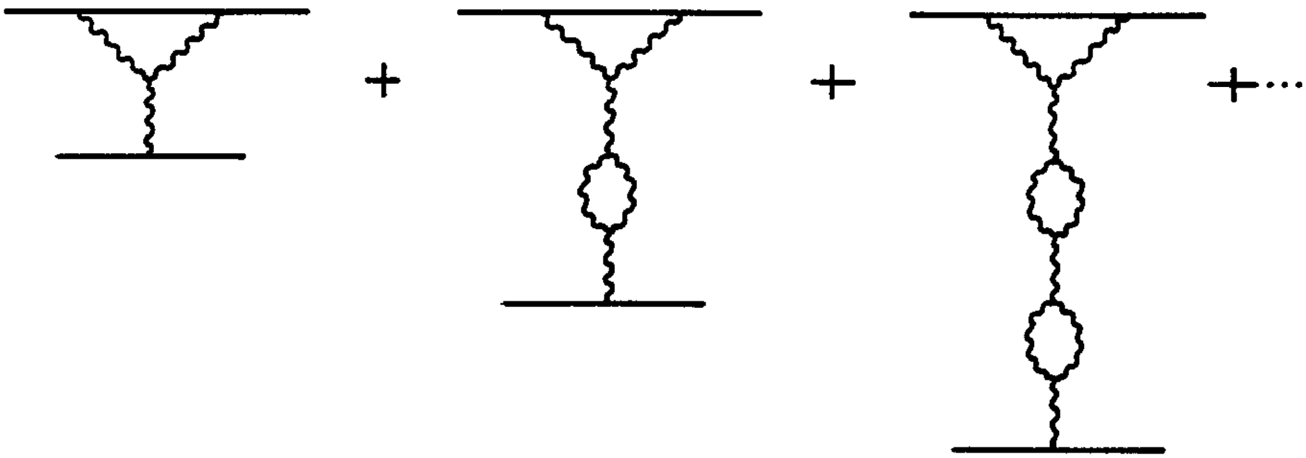
(a)



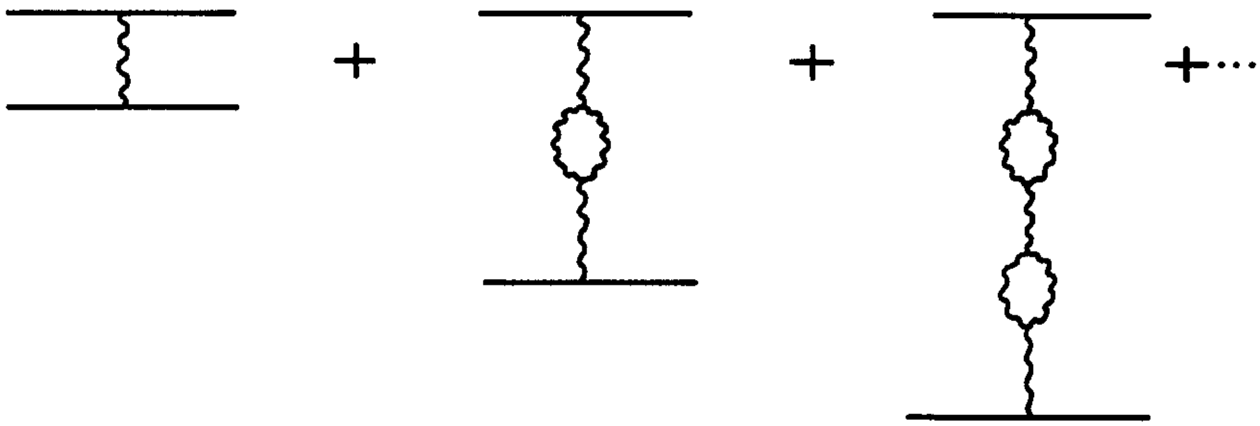
(b)



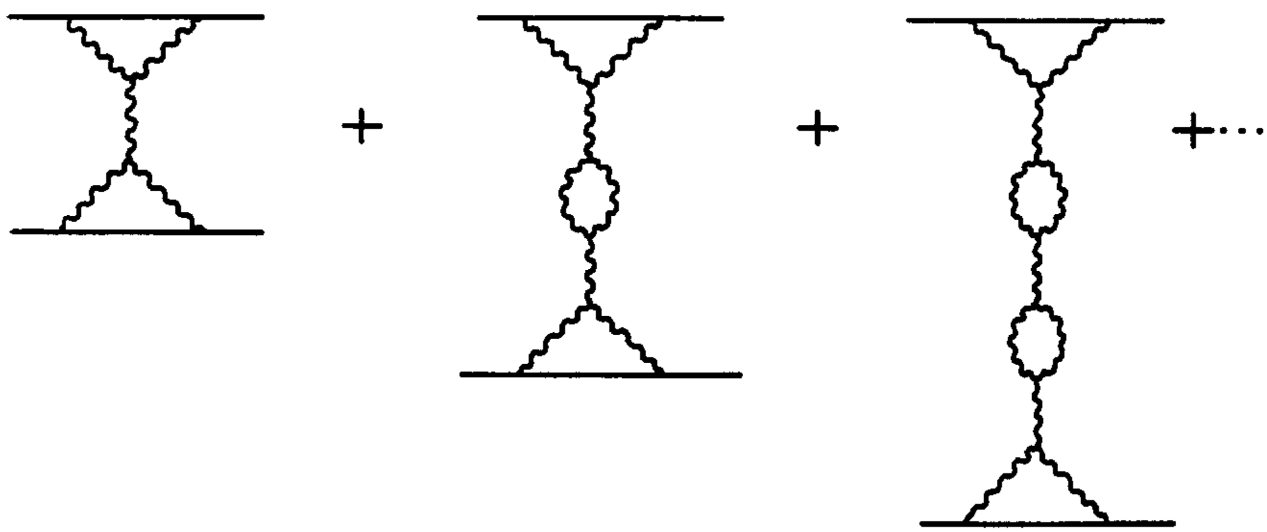
$\sim \beta^2 \; s$



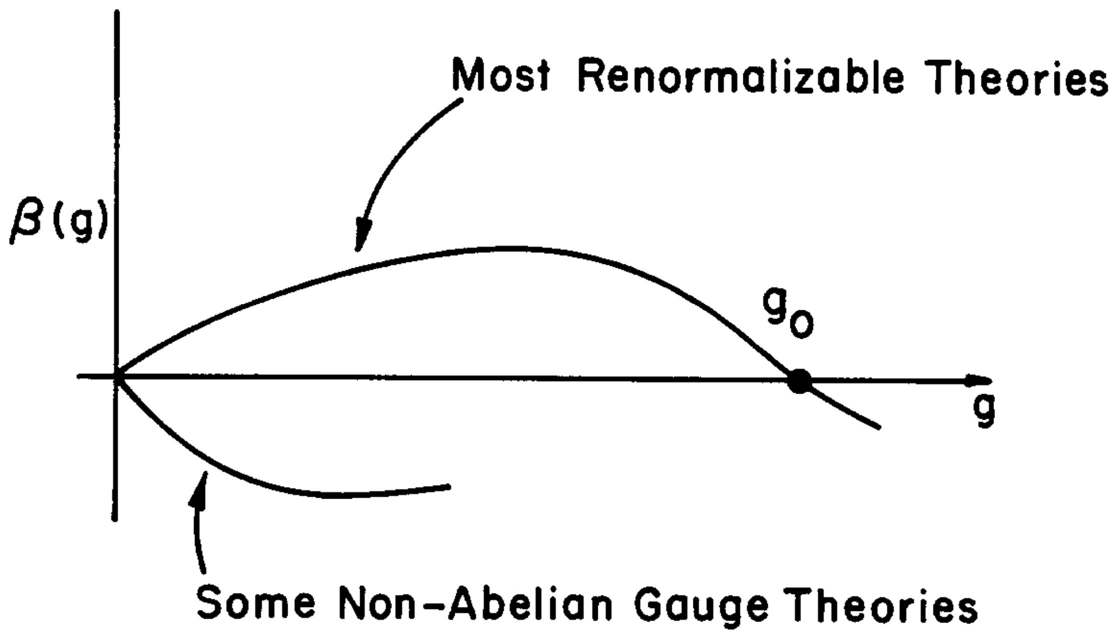
(a)

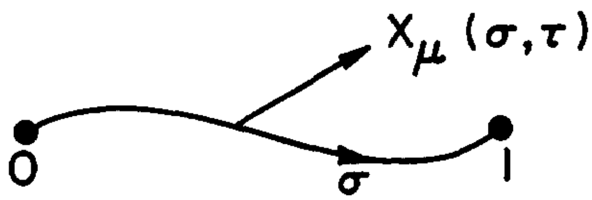


(b)

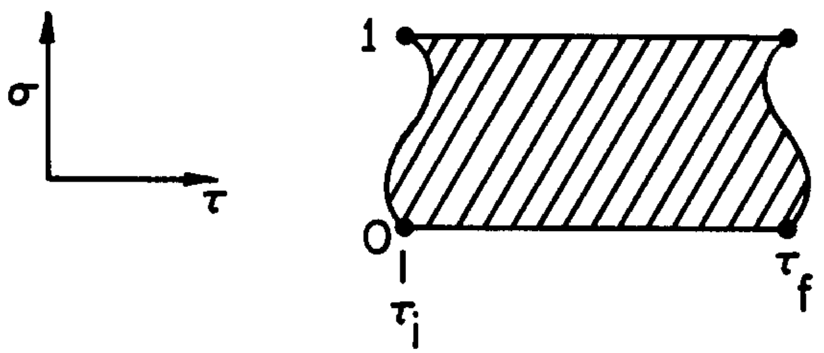


(c)


Most Renormalizable Theories
$\beta(g)$
g_0
g
Some Non-Abelian Gauge Theories



$X_\mu(\sigma,\tau)$
0
σ
1


σ
τ
1
0
τ_i
τ_f